On a class of graphs between threshold and total domishold graphs

A *total dominating set* in a graph is a subset of vertices such that every vertex in the graph has a neighbor in it. A graph is said to be *total domishold* if it admits a *total domishold structure*, that is, a hyperplane with non-negative coefficients that separates the characteristic vectors of its total dominating sets from the characteristic vectors of other vertex subsets. Hereditary total domishold graphs, that is, graphs every induced subgraph of which is total domishold, were recently characterized in terms of forbidden induced subgraphs; this characterization leads to an $O(|V(G)|^6)$ recognition algorithm of hereditary total domishold graphs.

The talk will be about a subclass $\mathcal{G}$ of the hereditary total domishold graphs obtained by replacing two of the forbidden induced subgraphs of order 6 with two of their proper induced subgraphs of order 5. We show that every connected graph in $\mathcal{G}$ is an extension of a threshold graph, that is, it can be obtained from some threshold graph G by attaching some pendant vertices to each vertex of G. Using this property, we develop a structural characterization of graphs in $\mathcal{G}$, which implies a linear time recognition algorithm for graphs in this class.

We also give a linear time algorithm for computing a total domishold structure of a graph in $\mathcal{G}$. In contrast to the algorithm for the general case of (hereditary) total domishold graphs, which relies on solving a linear program, the presented algorithm is purely combinatorial and works directly with the graph.

Joint work with Martin Milanič.