Some combinatorial aspects of the connections between hypergroups and fuzzy sets

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Hyperstructures theory and fuzzy sets theory are two traditional fields of theoretical and applied mathematics. The study of the connections between them has been an independent direction of research from the '70 of the past century, when A. Rosenfeld introduced the notion of fuzzy group. Later on, this concept was generalized to obtain new hypergroups and join spaces induced by fuzzy sets. Till now one distinguish three principal approaches of this theme: the study of new crisp hyperoperations obtained by means of fuzzy sets; the study of fuzzy subhyperstructures (i.e. fuzzy sets with the level sets being crisp hyperoperations); the study of structures endowed with fuzzy hyperoperations, called fuzzy hyperstructures.

In this survey talk I will concentrate on the first line of research. Its origin is due to P. Corsini, when he first defined a join space associated with a fuzzy set, and then a fuzzy set connected with a hypergroup. Iterating both constructions, a sequence of join spaces and fuzzy sets is obtained, that ends if two consecutive join spaces are isomorphic. The length of this sequence, i.e. the number of the non-isomorphic join spaces in it, is called the fuzzy grade of the initial hypergroup. At the beginning the sequence was studied for some particular hypergroups: the complete hypergroups and the hypergroups with partial scalar identities. Recently, I started to investigate this grade with a general and innovative method, making use of an ordered n-tuple determined by an equivalence relation.

A similar construction has been realized by Cristea and Davvaz using intuitionistic fuzzy sets. Although the fuzzy grade and the intuitionistic fuzzy grade have similar definitions, the corresponding associated sequences of join spaces have different properties. In this survey, the main goal is to realize a comparison between the both sequences.