A two-dimensional matrix problem of Molnár and Timmermann and its connection to a geometric problem of Rassias and Wagner

Our starting point is a result of L. Molnár and W. Timmermann concerning maps on the set of self-adjoint operators that preserves the operator-norm of the commutator ([4]). The problem remained open stubbornly in two-dimensions. The reason is that in an at least three dimensional, separable space we have the characterization of commutativity preserving maps (in both directions) which was provided by L. Molnár and P. Šemrl ([3, 6]).

It turned out that the above question is equivalent to the three-dimensional version of the following problem of T. M. Rassias and P. Wagner ([5]): characterize those mappings on a real Hilbert space that preserves the area of parallelograms spanned by any two vectors. We will present a solution to this question for general maps in finite dimensions and for bijections in general. Finally, I would like to discuss the corresponding problem concerning transformations preserving the k-dimensional volume of k-parallelepipeds spanned by any k vectors.

References

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