Characterization of vertex-transitive graphs of order pq

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Definition

Let $V = \mathbb{Z}_m \times \mathbb{Z}_n$*,* $\alpha \in \mathbb{Z}_n^*$*, and* $S_0, \ldots, S_m \subset \mathbb{Z}_n$ *such that* $\alpha^m S_i = S_i$*,* $i \in \mathbb{Z}_n$*. Define an* $(m, n, \alpha, S_0, \ldots, S_m)$-**metacirculant graph** $\Gamma = \Gamma(m, n, \alpha, S_0, \ldots, S_m)$ *by* $V(\Gamma) = \mathbb{Z}_m \times \mathbb{Z}_n$ *and* $E(\Gamma) = \{(\ell, j), (\ell + i, k)) : k - j \in \alpha^\ell S_i\}$.

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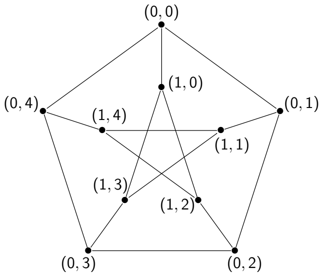

Figure : The Petersen graph is a $(2, 5, 2, \{1, 4\}, \{0\})$-metacirculant

Let $\alpha \in \mathbb{Z}_n^*$. Define $\rho, \tau : \mathbb{Z}_m \times \mathbb{Z}_n \mapsto \mathbb{Z}_m \times \mathbb{Z}_n$ by $\rho(i,j) = (i, j+1)$ and $\tau(i,j) = (i+1, \alpha j)$.

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A vertex-transitive digraph Γ of order qp, q and p distinct primes, is isomorphic to a (q, p)-metacirculant digraph if and only if $\mathrm{Aut}(\Gamma)$ contains a normal intransitive subgroup with orbits of size p.

Definition

Let G be a transitive group acting on X. A subset $B \subseteq X$ is a **block** *of G if whenever $g \in G$, then $g(B) \cap B = \emptyset$ or B. If $B = \{x\}$ for some $x \in X$ or $B = X$, then B is a* **trivial block**. *Any other block is nontrivial, and then G is* **imprimitive**. *If G is not imprimitive, we say that G is* **primitive**. *Note that if B is a block of G, then $g(B)$ is also a block of B for every $g \in G$, and is called a* **conjugate block of** *B. The set of all blocks conjugate to B, denoted $\mathcal{B}$, is a partition of X, and $\mathcal{B}$ is called a* **complete block system**.

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Definition

Let $q = 2^a$, *and* m *a divisor of* $q-1$. *Let* $S \subset \mathbb{Z}_m^*$ *such that* $-S = \{-s : s \in S\} = S$, *and* $U \subset \mathbb{Z}_m$. *Let* $w \in \mathbb{F}_q^*$ *generate* $\mathbb{F}_q^*$. *Then* $X(a, m, S, U)$ *is the graph with vertex set*

$$V(X(a, m, S, U)) = \mathrm{PG}(1, q) \times \mathbb{Z}_m,$$

the neighbors of $(\infty, r) \in V(X)$ *are*

$$\{(\infty, r+s) : s \in S\} \cup \{(y, r+u) : y \in \mathbb{F}_q, u \in U\},$$

and the neighbors of $(x, r) \in V(X)$, $x \in \mathbb{F}_q$ *are*

$$\{(x, r+s), (\infty, r-u), (x+w^i, -r+u+2i) : s \in S, i \in \mathbb{Z}_q, u \in U\}.$$

These graphs are the Marušič-Scapellato graphs.

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$\mathrm{SL}(2, 2^k)$ has m suborbits of length 1 and m suborbits of length 2^k. Additionally, if S is a suborbit of $\mathrm{SL}(2, 2^k)$ with respect to $x \in \infty$, then for every projective point $c \in \mathrm{PG}(1, 2^k)$ with $c \in \mathbb{F}_{2^k}$, we have $|S \cap c| = 1$.

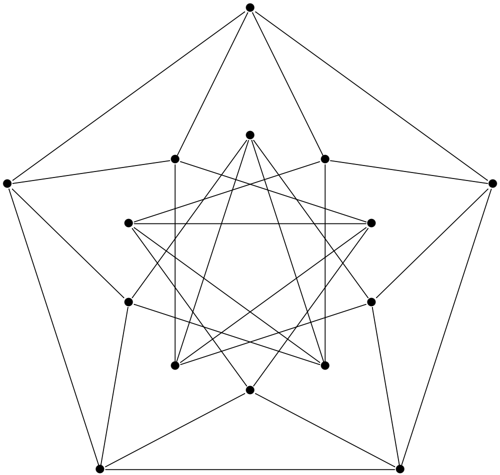

Figure : The line graph of the Petersen graph

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So, Marušič and Scapellato's classification is in terms of a minimal transitive subgroup of the automorphism group of the graph.

Table : Automorphism groups G of vertex-primitive graphs of order pq

pq	G	$\mathrm{soc}(G)$
pq	S^{pq}	A^{pq}
$p(p-1)/2$	S^p	A^p
$p(p+1)/2$	S^{p+1}	A^{p+1}
$(2^d \pm 1)(2^{d-1} \pm 1)$	$O^{\pm}(2d, 2)$	$\Omega^{\pm}(2d, 2)$
$(k+1)(k^2+1)$	$\mathrm{P\Gamma S}_p(4, k)$	$\mathrm{PSp}(4, k)$
$q(q^2+1)/2$	$\mathrm{P\Sigma L}(2, k^2)$	$\mathrm{PSL}(2, k^2)$
$p(p \pm 1)/2$	$\mathrm{PSL}(2, p)$	$\mathrm{PSL}(2, p)$
$3 \cdot 7$	$\mathrm{PGL}(2, 7)$	$\mathrm{PGL}(2, 7)$
$5 \cdot 7$	S^7	A^7
$5 \cdot 7$	$\mathrm{P\Gamma L}(4, 2)$	$\mathrm{PSL}(4, 2)$
$5 \cdot 11$	$\mathrm{PGL}(2, 11)$	$\mathrm{PGL}(2, 11)$
$7 \cdot 11$	$\mathrm{Aut}(\mathrm{M}_{22})$	M_{22}
$5 \cdot 31$	$\mathrm{P\Gamma L}(5, 2)$	$\mathrm{PSL}(5, 2)$
$7 \cdot 29$	$\mathrm{PSL}(2, 29)$	$\mathrm{PSL}(2, 29)$
$11 \cdot 23$	$\mathrm{PSL}(2, 23)$	$\mathrm{PSL}(2, 23)$
$29 \cdot 59$	$\mathrm{PSL}(2, 59)$	$\mathrm{PSL}(2, 59)$
$31 \cdot 61$	$\mathrm{PSL}(2, 61)$	$\mathrm{PSL}(2, 61)$
$3 \cdot 19$	$\mathrm{PSL}(2, 19)$	$\mathrm{PSL}(2, 19)$

Theorem

Let $G = \mathrm{PSL}(2, r^2)$ *act on the cosets of an isomorphic copy of* $\mathrm{PGL}(2, r)$ *in* G. *If* $r \equiv 1 \pmod 4$, *then* G *has*

- 1 *suborbit of length* $r(r-1)/2$,
- 1 *suborbit of length* $r^2 - 1$,
- $(r-5)/4$ *suborbits of length* $r(r-1)$,
- $(r-1)/4$ *suborbits of length* $r(r+1)$.

If $r \equiv 3 \pmod 4$, *then* G *has*

- 1 *suborbit of length* $r(r+1)/2$,
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Open Problems!

- Find full automorphism groups of Marušič-Scapellato graphs. Probably not a good problem for students here as will take detailed knowledge of the classification of the finite simple groups. But it's still an open problem.
- Complete the classification! What? Neither classification gives a way of determining whether a given graph *within* a class is being repeated. In other words, the isomorphism problem needs to be solved within each class. This was done a while back for metacirculant graphs, and I think recently for Marušič-Scapellato graphs. Other families remain, and to my knowledge no one has ever thought about those isomorphism problems (so they might be easy!).